Christ Church Grammar School

2018
TEST 2

SPECIALIST MATHEMATICS Year 12

Section One:
Calculator-free

Student name SOLUTIONS

Teacher name _____

Time and marks available for this section

Reading Time:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

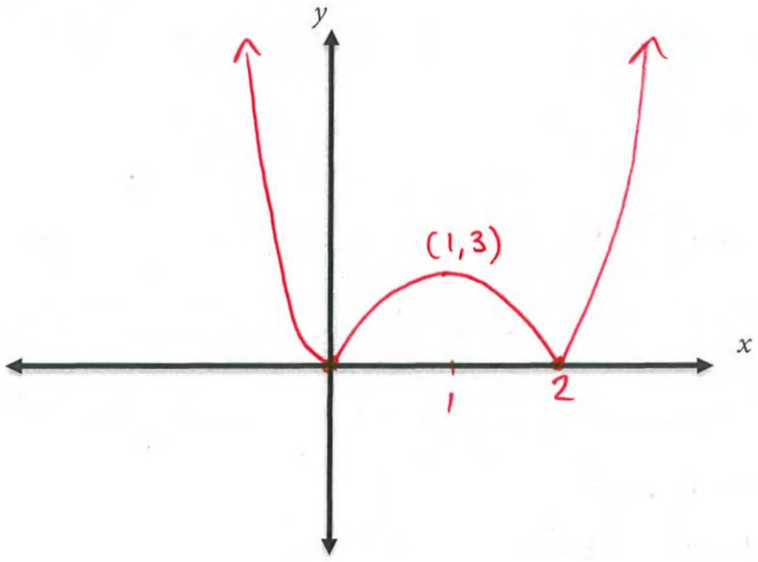
6. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(4 marks)**

If $f(x) = 3x(x-2)$, sketch the graphs

(a) $y = |f(x)|$ (2 marks)

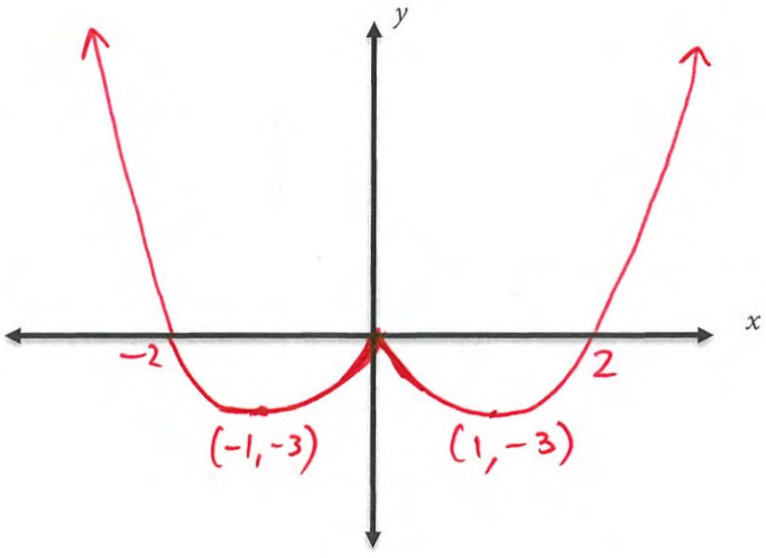


✓ for ∩

✓ for values on axes and ↑ ↑

Note: value of 1 or (1,3) not required as long as ∩ is symmetrical

(b) $y = f(|x|)$, (2 marks)



✓ for ∪ negative part of graph

✓ for ∪ and values on x-axis (TP not needed).

Question 2 **(3 marks)**

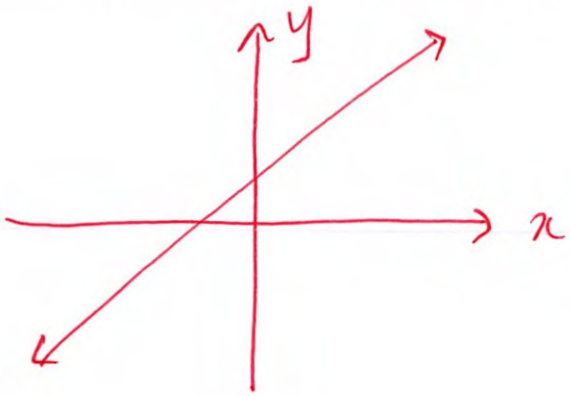
The horizontal line test says:
"For a function to have an inverse function, no horizontal line can cut its graph more than once:

(a) Explain why this is a valid test for the existence of an inverse function. (2 marks)

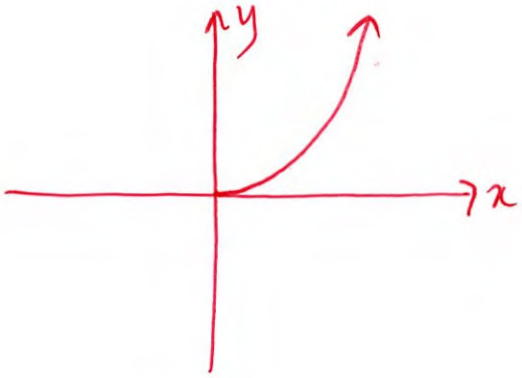
Inverse function must also be a function and must therefore satisfy the vertical line test, which it can only do if original if original function satisfies horizontal test. ✓ ✓

1 for inverse function statement
1 for satisfies vertical line test.

(b) Sketch an example of a function that would have an inverse. (1 mark)



✓ any function that passes horizontal test.



Question 3 **(2 marks)**

If $g\,o\,f(x) = \frac{x+2}{3x}$ and $g(x) = x - 2$.

Find the function defined as $f(x)$.

If $f(x) = q$ then $g(q) = q - 2$

and $g(q) = \frac{x+2}{3x}$

$\therefore \frac{x+2}{3x} = q - 2$

$q = \frac{x+2}{3x} + 2$

$= \frac{x+2+6x}{3x}$

$= \frac{7x+2}{3x}$

$\therefore f(x) = \frac{7x+2}{3x}$. ✓

✓ 1 for equating or simplifying expressions.

can get 2 marks for answer only.

See next page

Question 4 **(6 marks)**

Sketch the following graph

$$y = \frac{4x - 5}{x^2 - 1}$$

1 mark for these aspects; explicit or on graph

As $x \to 1^-$, $y \to \infty$

$x \to 1^+$, $y \to -\infty$

$x \to -1^-$, $y \to -\infty$

$x \to -1^+$, $y \to \infty$

$x \to \infty$, $y \to 0^+$

$x \to -\infty$, $y \to 0^-$

1 mark for VA on graph, must be labelled

VA at $x = \pm 1$

y-int: $x = 0$

$y = \frac{-5}{-1}$

$= +5$

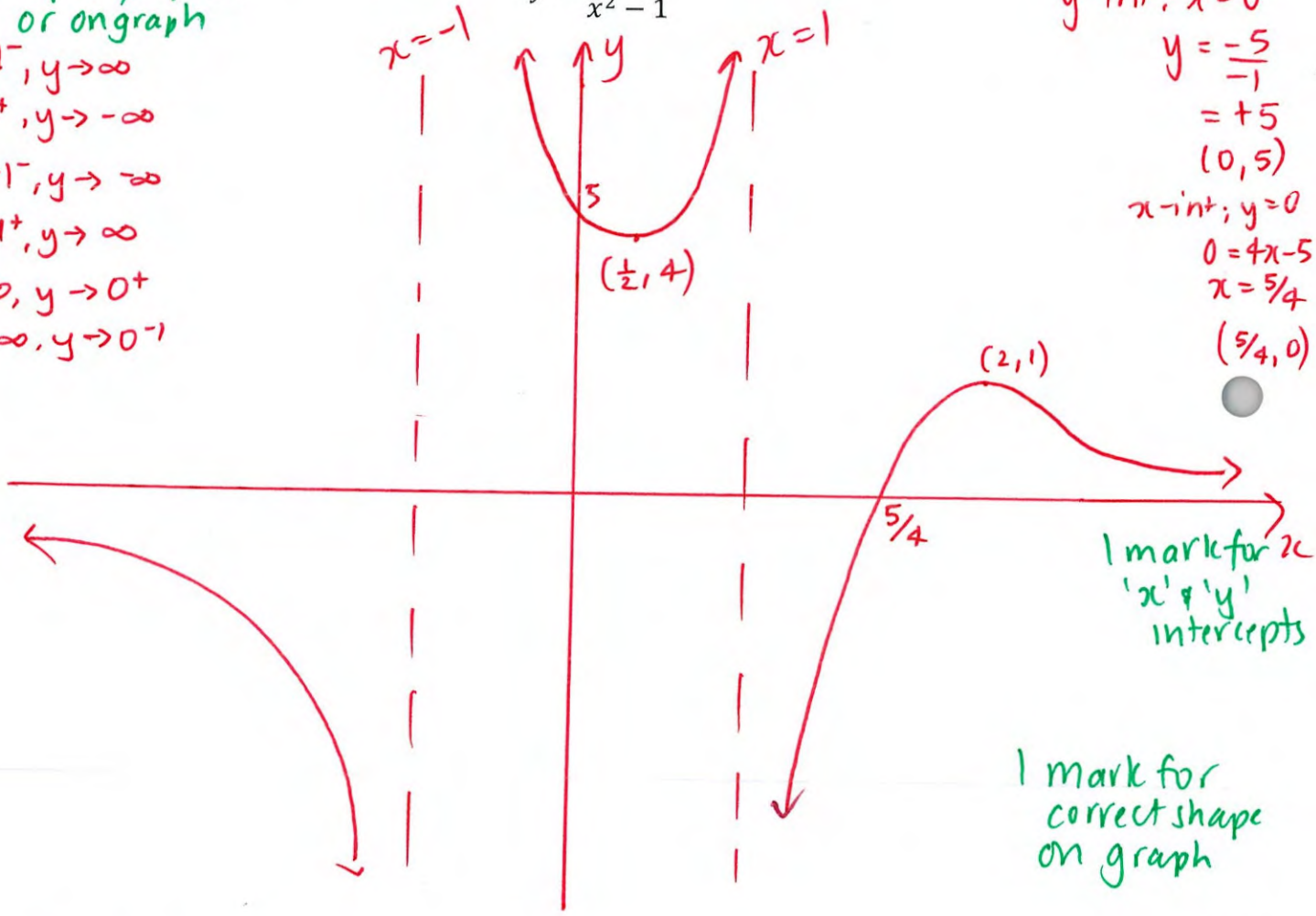
$(0, 5)$

x-int: $y = 0$

$0 = 4x - 5$

$x = 5/4$

$(5/4, 0)$



1 mark for 2c 'x' & 'y' intercepts

1 mark for correct shape on graph

$$y' = \frac{4(x^2-1) - 2x(4x-5)}{(x^2-1)^2}$$

$$= \frac{4x^2 - 4 - 8x^2 + 10x}{(x^2-1)^2}$$

$$= \frac{-4x^2 + 10x - 4}{(x^2-1)^2}$$

let $y' = 0$

$0 = -2(2x^2 - 5x + 2)$

$= (2x - 1)(x - 2)$

$\therefore x = \frac{1}{2}$ or 2

x	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	2	3
y'	−	0	+	+	0	−
	\	−	/	/	−	\

$\therefore$ min at $x = \frac{1}{2}$ max at $x = 2$

when $x = \frac{1}{2}$ $y = \frac{2-5}{\frac{1}{4}-1}$

$= \frac{-3}{-\frac{3}{4}}$

$= 4$

$x = 2$, $y = \frac{8-5}{4-1}$

$= 1$

√√ marks for $(2, 1)$ TP and nature

$(\frac{1}{2}, 4)$

Can use 2nd derivative test

End of questions

Christ Church Grammar School

2018
TEST 2

MATHEMATICS SPECIALIST Year 12

Section Two:
Calculator-assumed

Student name SOLUTIONS

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	31 minutes
Marks available:	31 marks

Materials required/recommended for this section

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Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

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See next page

Question 4 **(4 marks)**

Given that $f(x) = x^2 - 2$ and $g(x) = 3x + 1$

(a) Find the functions

(i) $k(x) = f(g(x))$ (1 mark)

$k(x) = (3x+1)^2 - 2$ ✓

$= 9x^2 + 6x - 1$ (accept as well, but not necessary)

(ii) $h(x) = g(f(x))$ (1 mark)

$h(x) = 3(x^2-2) + 1$

$= 3x^2 - 6 + 1$

$= 3x^2 - 5$ ✓

must simplify to this answer this time as terms are like.

(b) If $f(g(p)) = g(f(p)) + 16$, for some integer p.

Find the value(s) of p. (2 marks)

$(3p+1)^2 - 2 = 3p^2 - 5 + 16$ ✓ 1 for equating (letter must be 'p').

$p = -2$ or 1.

✓ 1 for solution (must have both)

Note: If answer only given award 2 marks as no working required for 2 mark qus.

If answer is $x = -2$ or 1 then only 1/2 as must recognise change of variable.

Question 5 **(3 marks)**

(a) Find the inverse $f^{-1}(x)$ of the function $f(x) = \frac{x+2}{2x-1}, x \neq \frac{1}{2}$ (1 mark)

$f^{-1}(x) = \frac{2+x}{2x-1}$ ✓

use calculator

or $\left(f^{-1}(x) = \frac{x+2}{2x-1}\right)$

(b) What does your solution from part (a) tell you about the function? (1 mark)

It is a self-inverse function ✓

(or symmetrical about the line $y=x$)

(c) State the domain for $f^{-1}(x)$. (1 mark)

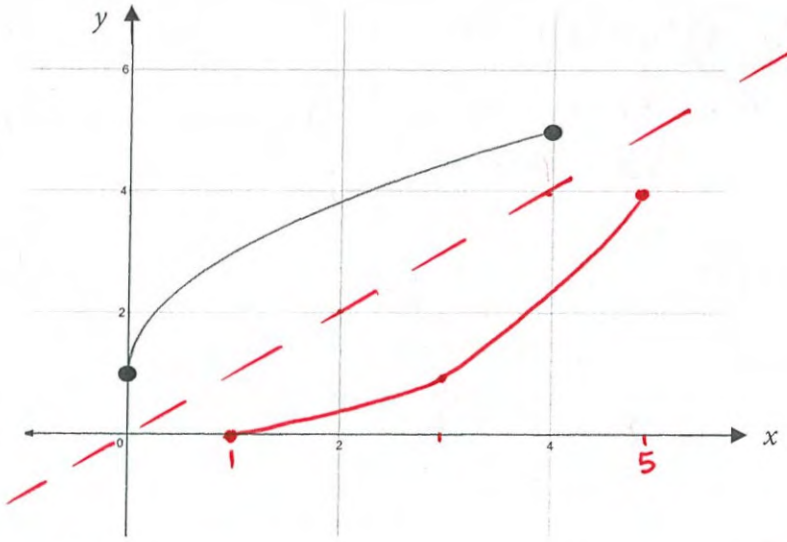
$\{x \in \mathbb{R} : x \neq \frac{1}{2}\}$ ✓

Note: accept $x \neq \frac{1}{2}$ but make comment to boys about how it should be written.

See next page

Question 6 **(5 marks)**

Shown below is the graph of $f(x) = 2\sqrt{x} + 1, \ for \ 0 \leq x \leq 4$



$y = x$

✓ correct end points (5,4) + (1,0)

✓ shape.

Note: ~~labels~~ points don't need to be labelled

(a) Plot the graph on $f^{-1}(x)$ on the same set of axes. (2 marks)

(b) Find an expression for $f^{-1}(x)$, clearly stating the domain and range for the inverse function. (3 marks)

$f(x) = 2\sqrt{x} + 1$ $\quad D: \{x \in \mathbb{R}: 0 \leq x \leq 4\}$

$R: \{y \in \mathbb{R}: 1 \leq y \leq 5\}$

$\therefore f^{-1}(x) = \frac{(x-1)^2}{4}$ ✓

$D: \{x \in \mathbb{R}: 1 \leq x \leq 5\}$ ✓

$R: \{y \in \mathbb{R}: 0 \leq y \leq 4\}$ ✓

Note: also accept $f^{-1}(x) = \frac{x^2}{4} - \frac{x}{2} + \frac{1}{4}$

or $f^{-1}(x) = \frac{1}{2}\left(\frac{x^2}{2} - x + \frac{1}{2}\right)$

or $f^{-1}(x) = \frac{1}{4}(x^2 - 2x + 1)$

Not acceptable $\left(\frac{x-1}{2}\right)^2$

See next page

Question 7 **(4 marks)**

$(x + 3)$ is a factor of $x^3 + ax^2 + bx + 24$ and when divided by $(x - 1)$, the remainder is 12. Find the values of a and b.

$(-3)^3 + a(-3)^2 + (-3)b + 24 = 0$

$-27 + 9a - 3b + 24 = 0$

$9a - 3b - 3 = 0$ (1)

1 for subst $x = -3$, $y = 0$ (can be simplified)

$(1)^3 + a(1)^2 + b(1) + 24 = 12$

$1 + a + b + 24 = 12$

$a + b + 13 = 0$ (2)

1 for subst $x = 1$, $y = 12$ (can be simplified)

Solve on calc $a = -3$ ✓

$b = -10$ ✓

1 for each solution

Note: can get final 2 marks if error in initial equations (only if you can see equations).

If write $a =$ $b =$ only, with no other working 2/4.

They must show some working to gain full marks as per dot point 5 on 2nd page of test.

See next page

Question 8 **(2 marks)**

Factorise $z^4 + 2z^3 - 2z^2 + 8$ into linear factors.

$(z+2)^2(z-1+i)(z-1-i)$ ✓✓

or $(z+2)^2(z-(1+i))(z-(1-i))$

on calculator so no working required. any order for brackets.

-1 for each error in bracket.

Question 9 **(5 marks)**

If $-2 + bi$ is a solution to $z^2 + az + (3 + a) = 0$.

Find a and b, in exact form, given that they are real.

$(-2+bi)^2 + a(-2+bi) + (3+a) = 0$ ✓ (for subst or simplifying)

$4 - 4bi - b^2 - 2a + abi + 3 + a = 0$

$-4bi - b^2 - a + abi + 7 = 0$

$\therefore$ Re: $-b^2 - a + 7 = 0$ ① $\Rightarrow a = 7 - b^2$ ✓

Im $-4b + ab = 0$ ② ✓

marks for equating Re = 0 Im = 0

$\therefore b = 0$ or $\pm\sqrt{3}$

Solve on calc.

If $b = 0$ $a = 7$ ✓

If $b = \pm\sqrt{3}$ $a = 4$ ✓

If solutions only then 2/5.

If solutions $b=0$ $a=7$ $b=\sqrt{3}$ $a=4$ then *1/5 or 4/5 if working.

Can get 4/5 if subst or simplification was wrong but everything else correct.

See next page

Question 10 (5/8 marks)

$$f(x) = \frac{x^2 - 6x + 14}{x - 1}$$

(a) Show algebraically, the graph of the the function

(i) Does not meet the x-axis (2 marks)

let $y = 0$ $\quad 0 = \frac{x^2 - 6x + 14}{x - 1}$

$0 = x^2 - 6x + 14$

cannot be solved ✓

$\therefore$ no real roots

1 for statement no real roots

$\Delta\ b^2 - 4ac$
$= (-6)^2 - 4 \times 1 \times 14$ ✓
$\Rightarrow$ (-)ve
$\therefore$ no real roots

evidence of not being able to be solved algebraically
Could complete square
$(x-3)^2 + 5 = 0$
$(x-3)^2 = -5$

(ii) Has no horizontal points of inflection but has two turning points (3 marks)

$f'(x) = \frac{x^2 - 2x - 8}{(x-1)^2}$ ✓

mark for finding $f'(x)$

let $f'(x) = 0 \Rightarrow 0 = \frac{x^2 - 2x - 8}{(x-1)^2}$

$x = 4$ or -2 ✓

mark for solving $f'(x) = 0$
(can you mark if $f'(x)$ is wrong)

x	-3	-2	0	3	4	5
$f'(x)$	+	0	-	-	0	+
	/	—	\	\	—	/
		max			min	

✓

evidence of finding TPs.
Can use $\frac{d^2y}{dx^2}$

$f''(x) = \frac{18}{(x-1)^3}$

$f''(4) > 0$ $\quad f''(-2) < 0$

$\therefore$ minimum $\quad \therefore$ maximum

See next page

Question 10 cont

(b) (i) Find the equation of the oblique asymptote (2 mark)

$$x-1 \overline{)\, x^2 - 6x + 14}$$

$x - 5$

$-(x^2 - x)$ ↓

$-5x + 14$

$-(-5x + 5)$

9

✓ Evidence to find asymptote

$$y = x - 5 + \frac{9}{x-1}$$

$\therefore$ oblique asymptote $y = x - 5$ ✓ If answer only award 2 marks (if ✓)

If no working & equation wrong 0/2
If working but equation wrong 1/2 if you can see error.

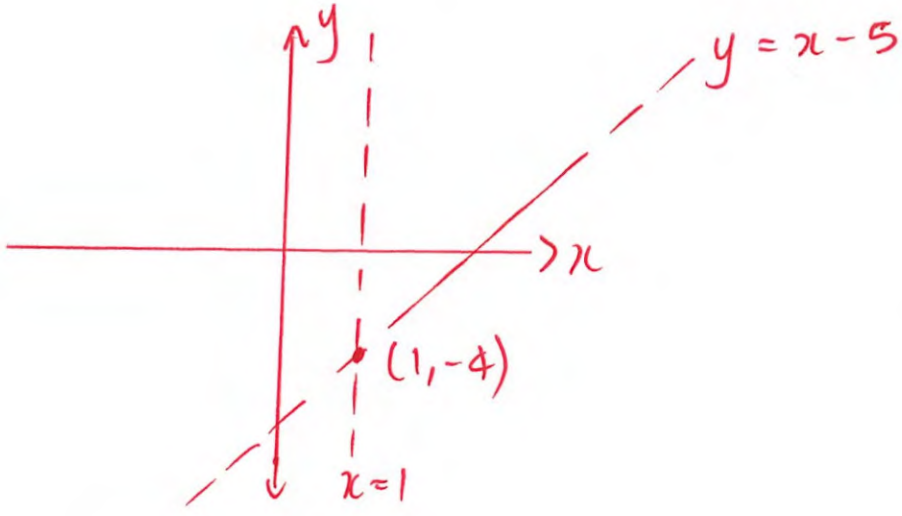
(ii) Show the two asymptotes of the function intersect at (1, -4) (1 mark)

VA $\Rightarrow x = 1$

$\therefore$ If $x = 1$ $y = 1 - 5$
$= -4$

✓ evidence that shows point of intersection.

$\therefore$ Intersect at $(1, -4)$



End of questions